

Categories of Physical Processes

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Part I
A non-topological TQFT

Construction Outline

Phys \longrightarrow ***Mod**

Construction Outline

- ▶ $*\mathbf{Mod}$ = representations of C^* -algebras + isometric relative homomorphisms

$\mathbf{Phys} \longrightarrow * \mathbf{Mod}$

$$H \xrightarrow{h} H' \quad (*\mathbf{Mod})$$

$$A \xrightarrow{f} B \quad (C^* \mathbf{Alg})$$

$$h(av) = f(a)h(v)$$

Construction Outline

Phys \longrightarrow $*\mathbf{Mod}$

$$\mathcal{S}(A) = \{\varphi : A \longrightarrow \mathbb{C}\}$$

φ positive

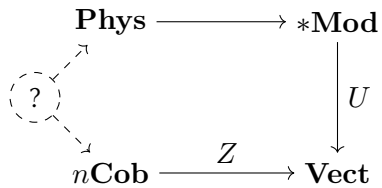
- ▶ $\mathcal{S} : C^* \mathbf{Alg}^{op} \longrightarrow \mathbf{Set}$
- ▶ **Phys** = pairs $(A, \varphi), \varphi \in \mathcal{S}(A)$
Phys = $1 \downarrow \mathcal{S}$
- ▶ \mathcal{S} monoidal \implies **Phys** monoidal
 $(A, \varphi) \otimes (B, \psi) = (A \otimes B, \varphi \otimes \psi)$

Construction Outline

$$\begin{array}{ccc} \mathbf{Phys} & \longrightarrow & \mathbf{*Mod} \\ \mathcal{O} \downarrow & & \\ \mathbf{C^*Alg}^{op} & & \end{array}$$

- ▶ $(A, \varphi) \mapsto A$
- ▶ “Noncommutative spaces”
- ▶ \mathcal{S} not Morita invariant

Construction Outline



- TQFT: what's the common ground?

Construction Outline

$$\mathbf{Phys} \xrightarrow{GNS} \mathbf{*Mod}$$

- ▶ What is *GNS*?

The GNS Construction

Definition

A pointed A -module (H, v) **represents** $\varphi : A \rightarrow \mathbb{C}$ if

$$\varphi(a) = \langle av, v \rangle_H$$

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Theorem (The Gelfand-Naimark-Segal Theorem)

- ▶ Positive φ have an **initial representation**
- ▶ A representation is initial iff it is cyclic
(cyclic = generated by the chosen vector)

Notation

- ▶ Initial representation of $\varphi = GNS(\varphi)$
- ▶ Representing vector = Ω
- ▶ Write H for (H, v)

The GNS Functor

H represents $\varphi \implies f^*H$ represents $f^*\varphi$

$$f^*H \longrightarrow H$$

$$B \xrightarrow{f} A \xrightarrow{\varphi} \mathbb{C}$$

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$GNS(f^*\varphi)$

$GNS(\varphi)$

$$B \xrightarrow{f} A \xrightarrow{\varphi} \mathbb{C}$$

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$$GNS(f^*\varphi) \quad f^*GNS(\varphi) \longrightarrow GNS(\varphi)$$

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H represents $\varphi \implies f^*H$ represents $f^*\varphi$

$$GNS(f^*\varphi) \xrightarrow{\exists!} f^*GNS(\varphi) \longrightarrow GNS(\varphi)$$

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Theorem

This gives a symmetric monoidal functor

$$\text{GNS} : \mathbf{Phys}^{op} \longrightarrow * \mathbf{Mod}$$

Proof.

Things exist by initiality. Diagrams commute by cyclicity. \square

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$$B \xrightarrow{f} A \xrightarrow{\varphi} \mathbb{C}$$

Theorem

This gives a symmetric monoidal functor

$$\text{GNS} : \mathbf{Phys}^{op} \longrightarrow * \mathbf{Mod}$$

It's going the wrong way!

The Covariant GNS Functor

Physically Correct Direction

$$\mathbf{Phys} \xrightarrow{GNS^{op}} *Mod^{op} \xrightarrow{\text{adjoint}} *Mod_{adj}$$

GNS_c

The diagram illustrates the relationship between three categories. On the left is the category \mathbf{Phys} . An arrow labeled GNS^{op} points from \mathbf{Phys} to the category $*Mod^{op}$. From $*Mod^{op}$, an arrow labeled "adjoint" points to the category $*Mod_{adj}$. A curved arrow labeled GNS_c points directly from \mathbf{Phys} to $*Mod_{adj}$.

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Definition

- ▶ $*Mod_{adj}$ is $*$ -modules with adjoint homomorphisms
- ▶ Adjoint homomorphisms: coisometries h such that

$$ah(v) = h(f(a)v)$$

- ▶ Equivalently: adjoints of homomorphisms

Part II
Physics From a Functor

The Schrödinger Picture – Example Factory

1. H – faithful A -module
2. $U : H \rightarrow H'$ – isometric linear map
3. $f : A \rightarrow B = UAU^*$ – algebra map given by $a \mapsto UaU^*$

Theorem (Lifting Schrödinger)

For any $\psi \in H$ we have $f : U\psi \rightarrow \psi \in \mathbf{Phys}$, and

$$\begin{array}{ccc} GNS(\psi) & \xrightarrow{GNS(f)} & GNS(U\psi) \\ \downarrow & & \downarrow \\ H & \xrightarrow{U} & H' \end{array}$$

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Corollary

If U is unitary, then $g(a) = U^*aU$ gives $g : \psi \rightarrow U\psi \in \mathbf{Phys}$, and

$$\begin{array}{ccc} GNS(\psi) & \xrightarrow{GNS_c(g)} & GNS(U\psi) \\ \downarrow & & \downarrow \\ H & \xrightarrow{U} & H' \end{array}$$

Symmetries and Unitary Representations

Why does a G -equivariant state give a unitary representation of G ?

$$G \longrightarrow \mathbf{Phys} \xrightarrow{GNS_c} * \mathbf{Mod}_{adj}$$

Symmetries and Unitary Representations

Why does a G -equivariant state give a unitary representation of G ?

Because of composition!

$$G \begin{array}{c} \xrightarrow{\quad} \mathbf{Phys} \xrightarrow{GNS_c} * \mathbf{Mod}_{adj} \\ \xrightarrow{\quad} * \mathbf{Mod}_{adj} \end{array}$$

Unitary representation of G !

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Bonus items:

- ▶ Groupoids of symmetries
- ▶ Equivariant GNS:

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$$\mathbf{Phys}^G \xrightarrow{GNS_c^G} * \mathbf{Mod}_{adj}^G \xrightarrow{U} \mathbf{Rep}(G)$$

- ▶ Compatibility with composite systems:

$$\varphi \otimes \psi \text{ has symmetry } G \times G'$$

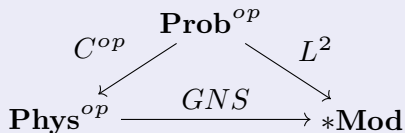
Relation to Probability Theory

Prob – compact probability spaces. From (X, μ) we construct:

- ▶ A state on $C(X)$ – the expectation value $\mathbb{E}_\mu(a) = \int_X a d\mu$
- ▶ $L^2(\mu)$, a $C(X)$ -module

Theorem

The following diagram of symmetric monoidal functors commutes



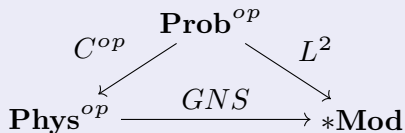
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Proof.

1. $L^2(\mu)$ is cyclic
2. $1 \in L^2(\mu)$ represents the expectation value \mathbb{E}_μ



Application: Eigenvalue-Eigenvector Link

Any normal $a \in \mathcal{O}(\varphi)$ determines a probability space

$$P_\varphi(a) = (\text{Spec}(\langle a \rangle), \varphi|_{\langle a \rangle})$$

Theorem (Eigenvalue-Eigenvector Link)

The following are equivalent:

1. $a\Omega = \lambda\Omega$
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Proof.

The inclusion $\langle a \rangle \subseteq \mathcal{O}(\varphi)$ gives a map $R : \varphi \rightarrow P_\varphi(a) \in \mathbf{Phys}$
Previous theorem computes $GNS(R)$:

$$L^2(\varphi|_{\langle a \rangle}) \rightarrow GNS(\varphi)$$

Thus: $a\Omega = \lambda\Omega \iff a \cdot 1 = \lambda \cdot 1$ in $L^2 \iff a = \lambda$ a.e. □

Classical Markov Processes

Definition (Markov Processes)

- ▶ $M(X)$ = probability measures on X
- ▶ Markov process $X \rightarrow Y = \text{map } X \rightarrow M(Y)$
- ▶ Category of Markov processes = $Kleisli(M)$

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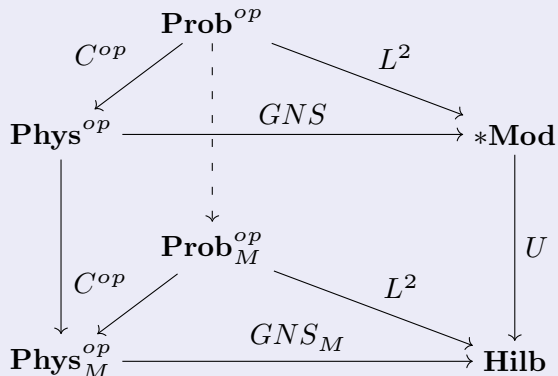
Theorem (Generalized Gelfand Duality; Furber & Jacobs 2015)

*Gelfand duality extends to a contravariant equivalence between Markov processes and **completely positive maps** between C^* -algebras*

Quantum Markov Processes

Theorem (Non-Unitary GNS Representation)

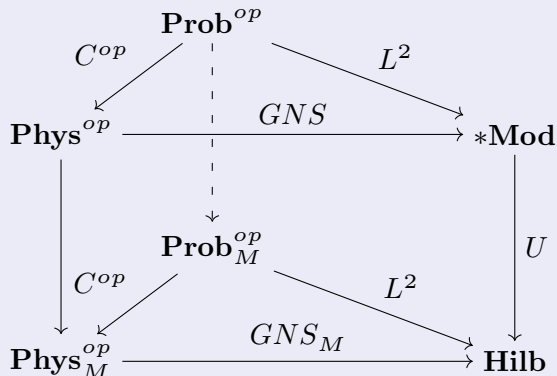
There is a commuting prism of symmetric monoidal functors:



Quantum Markov Processes

Theorem (Non-Unitary GNS Representation)

There is a commuting prism of symmetric monoidal functors:



$$GNS_{M,c} = (GNS_M)^*$$

Example: State Vector Collapse

- ▶ $P \in A$ – self-adjoint projection (= idempotent)
- ▶ $\Phi : A \rightarrow A$ given by $a \mapsto PaP$

Theorem

- ▶ φ represented by $\Omega \implies \Phi^*\varphi$ represented by $P\Omega$
- ▶ $GNS_M(\Phi)$ is the composite

$$GNS(\Phi^*\varphi) \hookrightarrow GNS(\varphi) \xrightarrow{P} GNS(\varphi)$$

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Corollary

$GNS_{M,c}(\Phi)$ is cyclic ($\Omega \mapsto P\Omega$), and acts as

$$GNS(\varphi) \xrightarrow{P} GNS(\varphi) \xrightarrow{\text{orth. proj.}} GNS(\Phi^*\varphi)$$

Interlude: The “Penrose Problem”

Naive Question

Is there a monoidal functor

$$n\mathbf{Cob} \longrightarrow \mathbf{Phys}_M,$$

which maps a non-unitary cobordism to a conditioning map?

Real Question

Dynamical spacetime represents a flux of information.

Can this information be used to condition state vectors?

Example: Particle Scattering

- ▶ H – Hilbert space
- ▶ $S : \mathcal{F}(H) \rightarrow \mathcal{F}(H)$ – unitary scattering matrix
- ▶ $H_\alpha, H_\beta \subseteq \mathcal{F}(H)$ – particles of type α and β

Proposition

There is a process $S_{\alpha\beta} : \alpha \rightarrow \beta \in \mathbf{Phys}_M$ such that

$$\begin{array}{ccccccc} H_\alpha & \xrightarrow{\text{inclusion}} & \mathcal{F}(H) & \xrightarrow{S} & \mathcal{F}(H) & \xrightarrow{\text{projection}} & H_\beta \\ & & & & & \searrow & \\ & & & & & & GNS_{M,c}(S_{\alpha\beta}) \end{array}$$

If you believe in QED:

$$\gamma + \gamma \rightarrow e^- + e^+$$

Part III

Stacks

What's a Stack?

Stack Semantics

Theorem

\mathcal{C} = superextensive site. Then:

$\mathbf{Stacks}(\mathcal{C})$ is a reflective localization of $\mathbf{Cat}(Sh(\mathcal{C}))$,

$$\begin{array}{ccc} & \text{“Stackify”} & \\ \mathbf{Cat}(Sh(\mathcal{C})) & \begin{array}{c} \xrightarrow{\quad} \\ \perp \\ \xleftarrow{\quad} \end{array} & \mathbf{Stacks}(\mathcal{C}) \\ & \text{“Split”} & \end{array}$$

inverting the *local equivalences*:

- ▶ Internal functors F such that $\vdash_{Sh(\mathcal{C})}$ “ F is an equivalence”

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Warning!

Size restrictions, universes, κ -ary superextensivity

What's a Stack?

Stack Semantics

- ▶ The language of $Sh(\mathcal{C})$ can describe internal categories
- ▶ Statements invariant under local equivalences make sense in $\mathbf{Stacks}(\mathcal{C})$ (variant of “stack semantics”)
- ▶ Idea: use this to make GNS a morphism of stacks

Internalizing $GNS : \mathbf{Phys}^{op} \longrightarrow *Mod$

Construction/Computation Steps

- ▶ Pick extension $\mathbb{A}_{\mathbb{R}}^1 \longrightarrow \mathbb{A}_{\mathbb{C}}^1$ with conjugation
- ▶ $*Mod$ – with bases (**flat**), nondegenerate hermitian forms
- ▶ Internal algebra: cyclic modules, bilinear forms
- ▶ \mathcal{S} = representable states (GNS theorem still nontrivial)
- ▶ Admissibility: not all maps in \mathbf{Phys} can be represented

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Remarks

- ▶ Sorely missed: reflective modules, **signed** infinitesimals
- ▶ Topoi easy, big sites hard (e.g. Zariski site)
- ▶ Examples: $Sh(X)$, $\mathbf{Flat}_{\mathbb{R}}$, models of SDG, ...

Definition

$\mathbf{Flat}_{\mathbb{R}}$ = schemes over \mathbb{R} and flat maps, with the Zariski topology

Infinitesimal Symmetries

1. G -equivariant $*$ -algebra $A : G \rightarrow * \mathbf{Alg}$
2. Compute $A(D)$ ($D =$ first order infinitesimals):

$$TG \rightarrow T \mathit{End}(A)$$

$$\mathit{Lie}(G) \rightarrow * \mathit{Der}(A)$$

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Theorem (GNS on Symmetry Generators)

If $X \in \text{Lie}(G)$ acts as $[Q, -]$ on A , and φ is G -equivariant, then in $\text{GNS}(\varphi)$:

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Morally true:

- ▶ $\text{GNS}_c(X) = -Q$
- ▶ Heisenberg \mapsto Schrödinger

The Classical Limit

Definition

1. \hbar -families = maps $\mathbb{A}^1 \rightarrow * \mathbf{Alg}$ or $\mathbb{A}^1 \rightarrow \mathbf{Phys}$
2. Classical limit = infinitesimal \hbar

Results on the Classical Limit

- ▶ It's a functor (e.g. preserves symmetries)
- ▶ $T_A(* \mathbf{Alg}) =$ deformations of A (i.e. HH^2) and possibly junk
- ▶ $\otimes =$ product of Poisson algebras (Hochschild cocycles)
- ▶ Singular spaces are quantized differently? More than Poisson!
E.g. isotropic connections, spacetimes with isometries
- ▶ Anomaly = obstruction to equivariant quantization?
- ▶ If $x(t)$ is a trajectory in a phase space X , then:

$$GNS(x(t)) = L^2(\delta_{x(t)}) \quad (\text{as } \mathcal{O}_X(X)\text{-modules})$$

Part IV

Things that aren't true yet

The Stack of Vacua

What this whole thing is for!

$\mathbf{Vac} \rightarrow \mathbf{Phys}^{\mathbf{Time}}$ stack of “minimal energy states”

- ▶ **Not a substack!** No obvious construction
- ▶ Energy is a $Z(A)$ -torsor \implies families of central observables destroy minimality
- ▶ Higher stack?

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- ▶ Higher stack?

What all this is for:

- ▶ **Path integral** = \hbar -connection on \mathbf{Vac} ?
- ▶ **Duality theorem** = equivalence in \mathbf{Vac} ?
- ▶ Moduli theory of vacua = differential geometry of

$$\mathbf{Vac} \xrightarrow{\mathcal{O}} * \mathbf{Alg}^{op}$$

- ▶ “Witten’s theorem”:
 $4d, N = 2$ super Yang-Mills has trivial \hbar -family of vacua
- ▶ RG flows = \mathbb{R}^* -equivariant families in \mathbf{Vac} ?

Higher Dimensional Misfits

- ▶ A_∞ observables: classical field theory uses E_∞
- ▶ What is Gauge Theory? Don't say "spacetime"!
- ▶ Locality according to Freed, Baez:
 1. *GNS* categorifies the partition function
 2. Does φ^4 define an extended QFT? Does QED?
 3. $n\mathbf{Phys}$ vs. $n\mathbf{Cob}$
- ▶ String Theory:
 1. Perturbatively a subcategory of \mathbf{Phys}
 2. Can "prove" it's not the whole story
 3. The landscape looks like a boundary

All essentially higher dimensional!

Thank You!